DETERMINING TIME OF A PHOTO GIVEN A DATE, LATITUDE, AND LONGITUDE

The problem itself conceptually is easy. The factors are (1) using the pictures to determine azimuth of the sun which needs true north, or the altitude of the sun which does not need true north. These would both be in degrees; altitude and azimuth both use latitude, and the date (for declination of the sun). Then inverting the formulae for altitude or azimuth, which gets the "lha" (local hour angle)(lha is 15 degrees per hour), and thus local time from noon, and with longitude plus the date (for the equation of time), correcting the result to legal time.

azimuth: the angle of the sun from true south. These notes have the formula for azimuth and it needs the suns declination (included along with a table)(this varies by day throughout the year from -23.44 to +23.44 degrees, 0 being the equinoxes and 23.55 being the solstices). The hour angle is what you want, you have already determined the azimuth. The hour angle is solar local time.

```
azi =  ATAN( SIN(lha) / ( (SIN(lat) * COS(lha)) – (COS(lat) * TAN(decl)))
yes              want          yes         want          yes        yes
```

the formula needs massaging for "lha", or you can put this in a spreadsheet and let it do increments for lha.

altitude: the angle of the sun from the pure horizon. These notes have the formula for altitude and it needs the suns declination (as above). The hour angle is what you want, you have already determined the altitude. The hour angle is solar local time.

```
alt = degrees(ASIN(SIN(decl)*SIN(lat) + COS(decl)* COS(lat)*COS(lha)))
yes                        yes       yes         yes         yes        want
```

the formula needs massaging for "lha", or you can put this in a spreadsheet and let it do increments for lha.

Determining azimuth or altitude from the pictures: the pictures are two dimensional. For azimuth, you have to draw the north south line through the base of a shadow casting device (gnomon), and measure the angle of the shadow from true north-south, and then estimate the real angle because the picture being 2d distorts it. For altitude, you measure the height of the shadow casting device in some unit, and the length of the shadow, in the same units. The altitude is atan(height/horizontal shadow length). With altitude or azimuth you can derive the lha (local hour angle of the sun, being 15 degrees per hour).

If you have both altitude and azimuth then you can cross check your work.

Once you have the lha (local hour angle), you have a time from local noon, this is ambiguous for morning and afternoon.

The difference between the longitude of the location and the longitude of the legal time zone in degrees is the correction for longitude, and at 4 minutes per degree you have (almost) legal time.

Finally, the sun varies by plus or minus 16 minutes approx, predictable through the year. So you correct for that and then you have the correct legal standard time.

Assumptions: the picture being 2d has lines drawn on it and the angles and heights or lengths are thus estimated. Further, consideration must be made for surfaces not being level or vertical. The declination of the sun uses formulae that are estimated, the table assumes on top of that a non leap year. The EOT (equation of time) similarly. The main spreadsheet on the web site does an astronomically accurate EOT, however remember that EOT does not vary by the day, but by the second.

SUNS ALTITUDE AND AZIMUTH ON ANY GIVEN HOUR GIVEN THE SUNS DECLINATION

The sun's declination is the value "decl" in the calculations above. Useful for the shepherd's dial.

ALTITUDE: The sun's altitude is its angle when looked at face on in degrees

alt = degrees(ASIN(SIN(decl) * SIN(lat) + COS(decl) * COS(lat)*COS(lha))) **A8.3**

AZIMUTH: azi = ATAN(SIN(lha) / ((SIN(lat) * COS(lha)) – (COS(lat) * TAN(decl)))

azi = degrees(atan(sin(RADIANS(15*(12-(hour/100))))/ (sin(radians(lat))*cos(RADIANS(15*(12-(hour/100))))- tan(radians(decl))*cos(radians(lat))))) [spreadsheet] **A8.4**

note: Some authors present two different formulae. They agree in all aspects except for 6am and thus also 6pm, when using the author's spreadsheet.

SUNS DECLINATION FOR ANY GIVEN DAY OF THE YEAR

Source: http://eande.lbl.gov/Task21/C2/algo1/1-11.html

Day number, J J=1 on 1 January, J=365 on 31 December. February is taken to have 28 days.

Jan	*Feb*	*Mar*	*Apr*	*May*	*Jun*
0	*31*	*59*	*90*	*120*	*151*
Jly	*Aug*	*Sep*	*Oct*	*Nov*	*Dec*
181	*212*	*243*	*273*	*304*	*334*

Day angle: da = 2 * pi * (j-1) / 365 (in radians, is an intermediate figure) **A8.1**

Sun Declination: decl = degrees (0.006918 – 0.399912*cos(da) + 0.070257*sin(da) – 0.006758*cos(2*da) + 0.000907*sin(2*da) – 0.002697*cos(3*da) + 0.001480*sin(3*da) **A8.2a**

alternative formula: DEGREES = (23.45*sin(radians(0.9678(jd-80)))) source: Claude Hartman **A8.2b**

SEE TABLE ON A SUBSEQUENT PAGE FOR DECLINATION BY DATE.

EQUATION OF TIME

A formula derived from Frans Maes from data by Savoie producing the EOT in minutes and using two sine waves is used for some spreadsheets, e.g. A2.1b, A2.1c. The values in the sin(...) function result in radians, so the formula is spreadsheet ready as-is. Value d = 1 to 365

A8.29a

E = 7.36*Sin(2*3.1416*(d-4.21)/365) + 9.92*Sin(4*3.1416*(d+9.9)/365)

Another formula using the sum of three sine waves is used for some spreadsheets, e.g. A2.1d, A2.1e. The sin(...) values result in degrees hence the required indicated radian conversion.

A8.29b

E = -1*(9.84*SIN(RADIANS(2*(360*(mm1+dd-81)/365))) –
7.53*COS(RADIANS(360*(mm1+dd-81)/365)) –
1.5*SIN(RADIANS(360*(mm1+dd-81)/365))) – 0.3

where: mm1 is the number of days prior to this month's day 1, So Jan is 0, Feb is 31, Mar is 59, April is 90, etc, assuming a non leap year. For leap years add 1 for March to December.

Jan	Feb	Mar	Apr	May	Jun	Jly	Aug	Sep	Oct	Nov	Dec
0	31	59	90	120	151	181	212	243	273	304	334

dd is the day of the month, being 1 to 31

i.e. mm1+dd is the Julian day of the year

Another three wave formula is:

A8.29c

E = 7.5*SIN(RADIANS(d-5)) – 10.2*SIN(RADIANS(1.93*(d-80))) +
0.5*SIN(RADIANS(1.5*(d-62)))

Every approximation is just that, and this book uses several methods for the EOT to demonstrate the real world of approximations, with their benefits as well as drawbacks. Even established published tables vary by almost a minute. Part of this is explained by the year within a leap year cycle, part by the decade the table was printed, and so on.

The most accurate formulae use the astronomical Julian day. This is somewhat involved and discussed in more detail in the earlier part of this book.

SEE TABLE ON A SUBSEQUENT PAGE FOR EQUATION OF TIME BY DATE.

SUN'S DECLINATION

A2.11

DECLINATION OF THE SUN BY THE DAY

A2.11 DECLINATION OF THE SUN BY DAY

	Jan	Feb	Mar	Apr	May	Jun	Jly	Aug	Sep	Oct	Nov	Dec
1	-23.1	-17.3	-7.9	4.2	14.8	21.9	23.2	18.2	8.6	-2.9	-14.2	-21.7
2	-23.0	-17.1	-7.5	4.6	15.1	22.1	23.1	18.0	8.2	-3.3	-14.5	-21.8
3	-22.9	-16.8	-7.1	5.0	15.4	22.2	23.0	17.7	7.8	-3.6	-14.8	-22.0
4	-22.8	-16.5	-6.7	5.4	15.7	22.3	23.0	17.5	7.5	-4.0	-15.1	-22.1
5	-22.7	-16.2	-6.3	5.8	16.0	22.5	22.9	17.2	7.1	-4.4	-15.5	-22.3
6	-22.6	-15.9	-6.0	6.2	16.3	22.6	22.8	16.9	6.7	-4.8	-15.8	-22.4
7	-22.5	-15.6	-5.6	6.5	16.6	22.7	22.7	16.6	6.4	-5.2	-16.1	-22.5
8	-22.3	-15.3	-5.2	6.9	16.9	22.8	22.6	16.4	6.0	-5.6	-16.4	-22.6
9	-22.2	-14.9	-4.8	7.3	17.1	22.9	22.5	16.1	5.6	-6.0	-16.7	-22.7
10	-22.1	-14.6	-4.4	7.7	17.4	23.0	22.4	15.8	5.2	-6.3	-16.9	-22.8
11	-21.9	-14.3	-4.0	8.0	17.7	23.0	22.2	15.5	4.9	-6.7	-17.2	-22.9
12	-21.8	-14.0	-3.6	8.4	17.9	23.1	22.1	15.2	4.5	-7.1	-17.5	-23.0
13	-21.6	-13.6	-3.2	8.8	18.2	23.2	22.0	14.9	4.1	-7.5	-17.8	-23.1
14	-21.4	-13.3	-2.8	9.1	18.4	23.2	21.8	14.6	3.7	-7.8	-18.0	-23.2
15	-21.3	-13.0	-2.4	9.5	18.7	23.3	21.7	14.3	3.3	-8.2	-18.3	-23.2
16	-21.1	-12.6	-2.0	9.8	18.9	23.3	21.5	14.0	3.0	-8.6	-18.6	-23.3
17	-20.9	-12.3	-1.6	10.2	19.1	23.4	21.3	13.7	2.6	-9.0	-18.8	-23.3
18	-20.7	-11.9	-1.3	10.5	19.4	23.4	21.2	13.4	2.2	-9.3	-19.1	-23.4
19	-20.5	-11.6	-0.9	10.9	19.6	23.4	21.0	13.0	1.8	-9.7	-19.3	-23.4
20	-20.3	-11.2	-0.5	11.2	19.8	23.4	20.8	12.7	1.4	-10.1	-19.5	-23.4
21	-20.1	-10.8	-0.1	11.6	20.0	23.5	20.6	12.4	1.0	-10.4	-19.8	-23.4
22	-19.9	-10.5	0.3	11.9	20.2	23.5	20.4	12.0	0.6	-10.8	-20.0	-23.4
23	-19.6	-10.1	0.7	12.3	20.4	23.5	20.2	11.7	0.2	-11.1	-20.2	-23.4
24	-19.4	-9.7	1.1	12.6	20.6	23.4	20.0	11.4	-0.1	-11.5	-20.4	-23.4
25	-19.2	-9.4	1.5	12.9	20.8	23.4	19.8	11.0	-0.5	-11.8	-20.6	-23.4
26	-18.9	-9.0	1.9	13.3	21.0	23.4	19.6	10.7	-0.9	-12.2	-20.8	-23.4
27	-18.7	-8.6	2.3	13.6	21.2	23.4	19.4	10.3	-1.3	-12.5	-21.0	-23.3
28	-18.4	-8.3	2.7	13.9	21.3	23.3	19.2	10.0	-1.7	-12.9	-21.2	-23.3
29	-18.2		3.1	14.2	21.5	23.3	18.9	9.6	-2.1	-13.2	-21.4	-23.3
30	-17.9		3.5	14.5	21.7	23.2	18.7	9.3	-2.5	-13.5	-21.5	-23.2
31	-17.6		3.9		21.8		18.5	8.9		-13.9		-23.1

DEGREES(0.006918 - 0.399912*COS(((2*3.1416*(jd-1)) / 365)) + 0.070257*SIN(((2*3.1416*
(jd-1)) / 365)) - 0.006758*COS(2*((2*3.1416*(jd-1)) / 365)) + 0.000907*SIN(2*((2*
3.1416*(jd-1)) / 365)) - 0.002697*COS(3*((2*3.1416*(jd-1)) / 365)) + 0.00148*SIN(3*
((2*3.1416*(jd-1)) / 365)))

DEGREES = (23.45*sin(radians(0.9678(jd-80))))   alternative formula agrees within half a degree

Different declination charts may disagree, factors affecting them would be leap year approximations, and the formula employed. Many formulae are approximations.

EOT ~ EQUATION OF TIME

minutes and seconds

GENERIC EOT TABLE

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1	3.11	13.30	12.31	4.06	-2.48	-2.18	3.41	6.22	0.17	-10.03	-16.26	-11.16
2	3.40	13.38	12.19	3.49	-2.56	-2.09	3.53	6.19	-0.02	-10.22	-16.27	-10.54
3	4.08	13.45	12.07	3.31	-3.02	-1.60	4.04	6.15	-0.21	-10.41	-16.28	-10.31
4	4.35	13.52	11.54	3.13	-3.09	-1.50	4.15	6.10	-0.41	-11.00	-16.28	-10.07
5	5.03	13.58	11.41	2.56	-3.14	-1.40	4.26	6.05	-1.01	-11.19	-16.28	-9.43
6	5.29	14.03	11.28	2.39	-3.19	-1.29	4.36	5.59	-1.21	-11.37	-16.26	-9.18
7	5.56	14.07	11.14	2.22	-3.24	-1.18	4.46	5.52	-1.41	-11.55	-16.24	-8.53
8	6.22	14.10	10.59	2.05	-3.28	-1.07	4.56	5.45	-2.01	-12.12	-16.21	-8.27
9	6.47	14.13	10.44	1.48	-3.31	-0.56	5.06	5.37	-2.22	-12.30	-16.17	-8.01
10	7.12	14.15	10.29	1.32	-3.34	-0.44	5.15	5.29	-2.42	-12.46	-16.12	-7.34
11	7.36	14.16	10.14	1.16	-3.36	-0.32	5.23	5.20	-3.03	-13.02	-16.06	-7.07
12	8.00	14.16	9.58	0.60	-3.38	-0.20	5.31	5.10	-3.24	-13.18	-15.59	-6.40
13	8.23	14.15	9.42	0.44	-3.39	-0.08	5.39	5.00	-3.45	-13.33	-15.52	-6.12
14	8.46	14.14	9.26	0.29	-3.40	0.05	5.46	4.50	-4.07	-13.48	-15.43	-5.44
15	9.08	14.12	9.09	0.14	-3.40	0.17	5.53	4.39	-4.28	-14.02	-15.34	-5.15
16	9.29	14.09	8.52	0.00	-3.39	0.30	5.59	4.27	-4.49	-14.16	-15.24	-4.46
17	9.50	14.05	8.35	-0.15	-3.38	0.43	6.05	4.15	-5.11	-14.29	-15.13	-4.17
18	10.10	14.01	8.18	-0.28	-3.37	0.56	6.10	4.02	-5.32	-14.41	-15.01	-3.48
19	10.29	13.56	8.00	-0.42	-3.34	1.09	6.14	3.49	-5.53	-14.53	-14.49	-3.19
20	10.48	13.50	7.43	-0.55	-3.32	1.22	6.18	3.35	-6.15	-15.05	-14.35	-2.49
21	11.06	13.44	7.25	-1.07	-3.28	1.35	6.22	3.21	-6.36	-15.15	-14.21	-2.19
22	11.23	13.37	7.07	-1.20	-3.24	1.48	6.25	3.06	-6.57	-15.25	-14.06	-1.50
23	11.39	13.29	6.49	-1.31	-3.20	2.01	6.27	2.51	-7.19	-15.35	-13.50	-1.20
24	11.54	13.21	6.31	-1.43	-3.15	2.14	6.29	2.35	-7.40	-15.43	-13.34	-0.50
25	12.09	13.12	6.13	-1.54	-3.10	2.27	6.30	2.19	-8.01	-15.51	-13.16	-0.20
26	12.23	13.03	5.55	-2.04	-3.04	2.40	6.31	2.03	-8.21	-15.58	-12.58	0.09
27	12.36	12.52	5.37	-2.14	-2.57	2.52	6.31	1.46	-8.42	-16.05	-12.39	0.39
28	12.49	12.42	5.19	-2.23	-2.50	3.05	6.31	1.29	-9.02	-16.10	-12.19	1.08
29	13.00		5.00	-2.32	-2.43	3.17	6.30	1.11	-9.23	-16.15	-11.59	1.38
30	13.11		4.42	-2.40	-2.35	3.29	6.28	0.53	-9.43	-16.20	-11.38	2.07
31	13.21		4.24		-2.27		6.25	0.35		-16.23		2.36

If "+" then add to solar time to get mean time as the sun is slow. If " –" then subtract from solar time to get mean time as the sun is fast. Some tables have a plus for our minus and vice versa. If in doubt look at the figure of eight equation of time. This table is based on the astronomical formulae and used 2007 as its basis, not a leap year. **Formulae involving dates** use approximations thus these tables may disagree with other sources using other formulae. This and other publications have figures that are well within drafting tolerances.

LOCAL HOUR ANGLE ~ "lha"

Sun's apparent hour angle			LHA=local hour angle							
Time		From	LHA	Radians	sin	cos	tan	cot	PM	PM
hhmm	hh.hh	midday							hh.hh	hhmm
4.00	4.00	8.00	120.00	2.0944	0.8660	-0.5000	-1.7321	-0.5774	8.00	8.00
4.15	4.25	7.75	116.25	2.0289	0.8969	-0.4423	-2.0278	-0.4931	7.75	7.45
4.30	4.50	7.50	112.50	1.9635	0.9239	-0.3827	-2.4142	-0.4142	7.50	7.30
4.45	4.75	7.25	108.75	1.8980	0.9469	-0.3214	-2.9459	-0.3395	7.25	7.15
5.00	5.00	7.00	105.00	1.8326	0.9659	-0.2588	-3.7321	-0.2679	7.00	7.00
5.15	5.25	6.75	101.25	1.7671	0.9808	-0.1951	-5.0273	-0.1989	6.75	6.45
5.30	5.50	6.50	97.50	1.7017	0.9914	-0.1305	-7.5958	-0.1317	6.50	6.30
5.45	5.75	6.25	93.75	1.6362	0.9979	-0.0654	-15.2571	-0.0655	6.25	6.15
6.00	6.00	6.00	90.00	1.5708	1.0000	0.0000	inf	0.0000	6.00	6.00
6.15	6.25	5.75	86.25	1.5053	0.9979	0.0654	15.2571	0.0655	5.75	5.45
6.30	6.50	5.50	82.50	1.4399	0.9914	0.1305	7.5958	0.1317	5.50	5.30
6.45	6.75	5.25	78.75	1.3744	0.9808	0.1951	5.0273	0.1989	5.25	5.15
7.00	7.00	5.00	75.00	1.3090	0.9659	0.2588	3.7321	0.2679	5.00	5.00
7.15	7.25	4.75	71.25	1.2435	0.9469	0.3214	2.9459	0.3395	4.75	4.45
7.30	7.50	4.50	67.50	1.1781	0.9239	0.3827	2.4142	0.4142	4.50	4.30
7.45	7.75	4.25	63.75	1.1126	0.8969	0.4423	2.0278	0.4931	4.25	4.15
8.00	8.00	4.00	60.00	1.0472	0.8660	0.5000	1.7321	0.5774	4.00	4.00
8.15	8.25	3.75	56.25	0.9817	0.8315	0.5556	1.4966	0.6682	3.75	3.45
8.30	8.50	3.50	52.50	0.9163	0.7934	0.6088	1.3032	0.7673	3.50	3.30
8.45	8.75	3.25	48.75	0.8508	0.7518	0.6593	1.1403	0.8770	3.25	3.15
9.00	9.00	3.00	45.00	0.7854	0.7071	0.7071	1.0000	1.0000	3.00	3.00
9.15	9.25	2.75	41.25	0.7199	0.6593	0.7518	0.8770	1.1403	2.75	2.45
9.30	9.50	2.50	37.50	0.6545	0.6088	0.7934	0.7673	1.3032	2.50	2.30
9.45	9.75	2.25	33.75	0.5890	0.5556	0.8315	0.6682	1.4966	2.25	2.15
10.00	10.00	2.00	30.00	0.5236	0.5000	0.8660	0.5774	1.7321	2.00	2.00
10.15	10.25	1.75	26.25	0.4581	0.4423	0.8969	0.4931	2.0278	1.75	1.45
10.30	10.50	1.50	22.50	0.3927	0.3827	0.9239	0.4142	2.4142	1.50	1.30
10.45	10.75	1.25	18.75	0.3272	0.3214	0.9469	0.3395	2.9459	1.25	1.15
11.00	11.00	1.00	15.00	0.2618	0.2588	0.9659	0.2679	3.7321	1.00	1.00
11.15	11.25	0.75	11.25	0.1963	0.1951	0.9808	0.1989	5.0273	0.75	0.45
11.30	11.50	0.50	7.50	0.1309	0.1305	0.9914	0.1317	7.5958	0.50	0.30
11.45	11.75	0.25	3.75	0.0654	0.0654	0.9979	0.0655	15.2571	0.25	0.15
12.00	12.00	0.00	0.00	0.0000	0.0000	1.0000	0.0000	inf	0.00	0.00

REFERENCE MATERIAL

a4.1	altitude spreadsheet	**a4.1 suns altitude.xls**
a4.2	azimuth spreadsheet	**a4.2 suns azimuth.xls**
	main spreadsheet	**illustrating-shadows.xls**
	astronomical EOT	**illustrating-shadows-eot.xls**

www.illustratingshadows.com

October 28, 2008