REFLECTING DIALS (ALSO KNOWN AS CEILING DIALS)

Ceiling dials are not common, Sir Isaac Newton and Christopher Wren both built them, and having an unused ceiling in my barn, I decided to follow suit.


The dial has hour lines, a meridian line depicted by sun declination markers, and the equinox and solstice curves. The pictures above show the time approaching 1 pm.

Having a habit of forgetting key information, a data plate shows the latitude and longitude of design, the declination (why the meridian is offset), and the distance from the ceiling to the mirror (CTM).

With a reflecting dial, or ceiling dial, a mirror is placed on a surface such as a window sill, of course it is set level. The sun is reflected from the mirror at point "m", and for this example, the equinox is assumed. The reflected ray displays a reflection of the sun on the ceiling, a spot, and this is at point "n". Because the following uses the equinox, angle "Cmq" is a right angle.
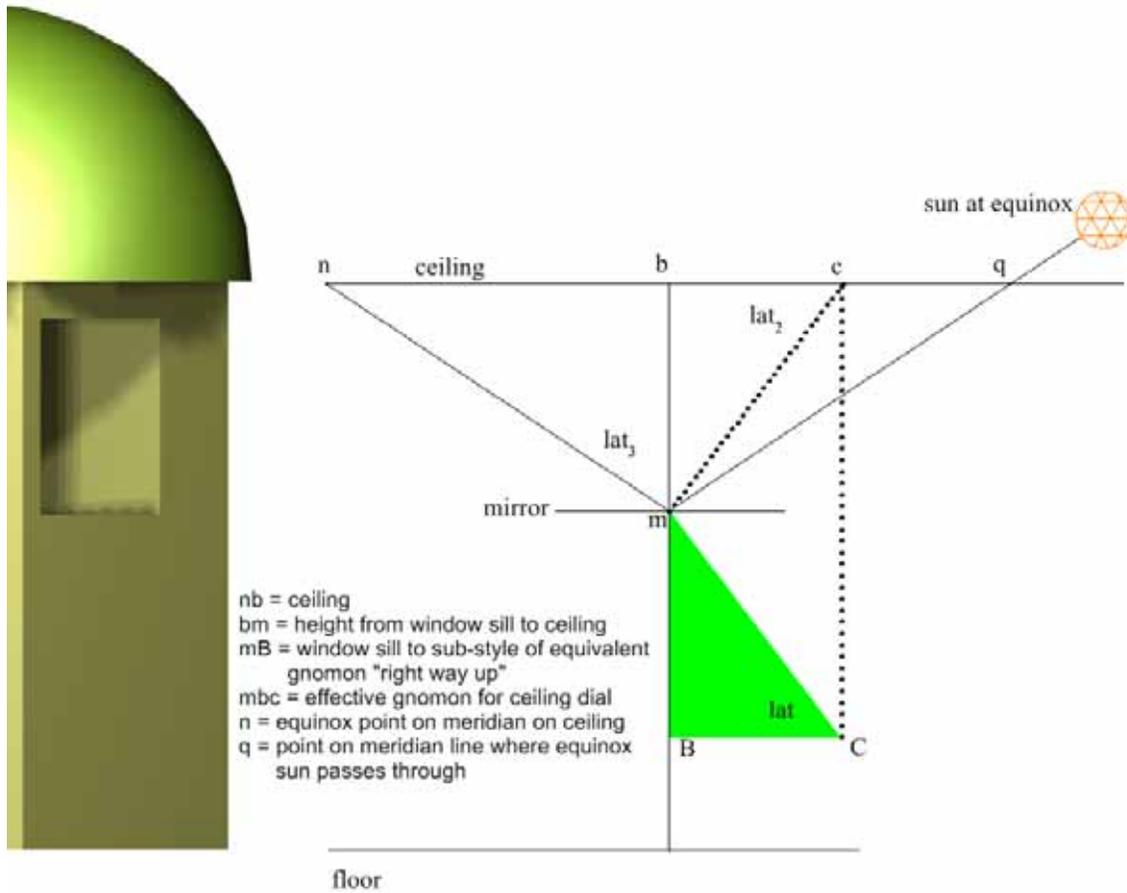
The sun comes through "q" to "m" and is reflected to "n". If this were a true horizontal dial, the gnomon would be "BmC", where "Bm" is the style linear height, and this matches the height of the ceiling above the little mirror, "mb". (NOTE: lat = lat_2 = lat_3, subscripts are order of derivation).

Angle "BCm" is the latitude, so also is "bcm" and "nmb".

Since nmb = latitude,   tan (lat) = nb / bm   thus   nb = mb * tan(lat)   [1]

Since bcm= latitude,   tan (lat) = mb / bc   thus   bc = mb / tan (lat)

thus, distance from equinox point on meridian
to the dial canter "c" is   nc = nb + bc   hence   nc = mb * tan(lat) + mb / tan(lat)
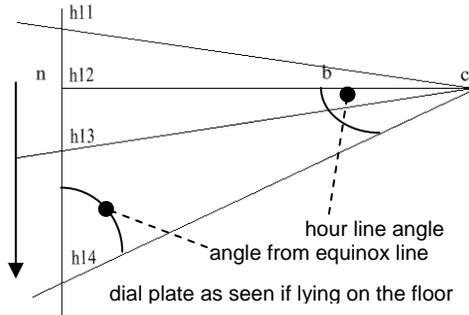
$$= mb * (tan(lat) + 1/tan(lat))   [2]$$
[The conventional formula is = 2*mb /sin(2*lat)]

Thus we have a dial plate, with a meridian line (L.A.T. noon) and an equinox point on it, thus the equinox line can be drawn.

Similarly, we have the dial center, it is located from the equinox point "n" at a distance:-

$$= mb * (\ tan(lat) + 1/(tan(lat) \ ) \qquad \text{derived formula} \qquad [\ 2\ ]$$

dial plate as seen if lying on the floor

The dial plate on the ceiling is reversed, otherwise it is a normal horizontal dial. The morning hours are to our right as we look up at the ceiling, whereas looking down on a normal horizontal dial, they are to the left. With this in mid, the dial layout is a normal horizontal dial for the design latitude and longitude, except we reverse the sense of the angles.

Ceiling dials have a dial center that is not in existence, but it is used to locate the hour line angles. For example, 2pm, the "h14" line will have an hour line angle of some value "x". Since the dial center "c" is not in physical existence, we use the distance from point "n", hour point on equinox line h14 from "n"

$$nh_{14} \quad = \quad tan(\text{hour line angle}) *nc$$

$$= \quad tan(\text{hour line angle}) * mb * (\ tan(lat) + 1/(tan(lat) \ )$$

and at point h14, a line is drawn of 90-hour line angle. Hence the hour lines can be drawn. Calendar curves can also be drawn using the same techniques covered elsewhere. A key point to remember is that these dials can get rather large, reducing the mirror's height from the ceiling will make the dial smaller.

REFLECTING OR CEILING DIALS

Distance from mirror to ceiling (style linear height)

1	nb = mb * tan(lat) [1]	nc=mb*(tan(lat)+1/tan(lat)) [2]
Latitude	Distance from point above mirror to equinox point on the meridian line	Distance from meridian equinox point to dial center
30	0.5774	2.3094
31	0.6009	2.2651
32	0.6249	2.2252
33	0.6494	2.1893
34	0.6745	2.1571
35	0.7002	2.1284
36	0.7265	2.1029
37	0.7536	2.0806
38	0.7813	2.0612
39	0.8098	2.0447
40	0.8391	2.0309
41	0.8693	2.0197
42	0.9004	2.0110
43	0.9325	2.0049
44	0.9657	2.0012
45	1.0000	2.0000

Latitude	Distance from point above mirror to equinox point on the meridian line	Distance from meridian equinox point to dial center
46	1.0355	2.0012
47	1.0724	2.0049
48	1.1106	2.0110
49	1.1504	2.0197
50	1.1918	2.0309
51	1.2349	2.0447
52	1.2799	2.0612
53	1.3270	2.0806
54	1.3764	2.1029
55	1.4281	2.1284
56	1.4826	2.1571
57	1.5399	2.1893
58	1.6003	2.2252
59	1.6643	2.2651
60	1.7321	2.3094
61	1.8040	2.3584

First, a place was estimated about where to place a mirror. Then in this case a cross piece was affixed to both sides of a window frame. This was level in all exes.

Then a point on the ceiling was selected, and a small hook located there, and a plumb line dropped.

A small piece of wood was placed on the cross piece, and a reflective surface added. Again, it was leveled in all axes.

The distance from the reflective surface to the ceiling was measured and found to be 48.5 inches.

The illustrating-shadows.xls spreadsheet was run with the design latitude of 32.75 and longitude of 108.2, and the vertical distance of 48.75 entered.

The date was November 30, 2008, the sun was fast so the EOT was negative, being – 10:59 minutes : seconds

Also, the sun would be slow due to the longitude separation from the legal meridian, and 108.2 – 105.0 is some 3.2 degrees, or +12:48 minutes : seconds

The total correction being: + 1:48 minutes / seconds

So at 12:00, plus the final correction of +1:48 minutes/secs, the light spot is finally marked on the ceiling at 1201:48 and then a line was drawn from that spot light to the point directly above the mirror, this was the noon meridian line.

The spreadsheet estimated a distance from the point above the mirror, to the light spot for the 11/30/08 date using the calculated sun's declination of -21.69 degrees as 67.85 inches. The actual distance was measured at 67.0" thus the calculations were matching the observations within less than an inch.

NOTE: The correction for longitude and EOT on the spreadsheet shows 2.39 on the next page, this is because the spreadsheet uses the simple EOT formula, and not the astronomical one used in the actual calculations above.

The main spreadsheet has all the data on the first section, see immediately below.

48.5	Distance from mirror to ceiling taken from main spreadsheet gnomon linear height				
			Todays EOT/Diff.Long correction:		2.39
	nb = mb * tan(lat)	nc=mb*(tan(lat)+1/tan(lat))			
Latitude	Distance from point above mirror to equinox point on the meridian line	Distance from meridian equinox point to dial center	Distance from point above mirror to summer solstice point on the meridian line	Distance from point above mirror to **today**'s decl point on the meridian line	Distance from point above mirror to winter solstice point on the meridian line
32.75	31.1962	106.5979	7.951	67.847	72.421
				Today's decl	
DESIGN LONGITUDE		**108.20**		-21.6913	
DESIGN LEGAL MERIDIAN		**105.00**		11/30/08	

Then the summer solstice, equinox, and winter solstice points were marked, they were derived from the spreadsheet as: 8 inches (7.951), 31.2 inches (311962), and 72.4 inches (72.421) respectively.

A line perpendicular to the meridian line was drawn at the equinox point on the meridian line, being at 31.2 inches.

The distance from the meridian line, along the equinoctial line for hour points and, repeated, are the inverse hour line angles at those hour points,			
Latitude	hour line angle at the hour point on the equinoctial line on the meridian ~ taken from above inverse hour line angles	Distance from meridian equinox point to dial center	Distance along equinoctial line from meridian line
32.75		106.5979	
am to pm	angle		
6.0	5.90		1031.446
7.0	21.12		276.035
8.0	43.04		114.161
9.0	58.82		64.497
10.0	70.51		37.736
11.0	79.91		18.960
12.0	88.27		3.224
13.0	83.55		12.047
14.0	74.72		29.130
15.0	64.19		51.560
16.0	50.42		88.124
17.0	31.29		175.394
18.0	5.90		1031.446

The last section of the spreadsheet identifies hour line distances along the equinoctial line from the meridian. It also copies the inverse hour line angles at those hour points.

The hour point along the equinox were marked.

At those hour line points, the inverse hour line angles were drawn.

Then the hour lines themselves can be completed, they would meet at the dial center which is of course outside the building.

The equinox line is used since it is straight. Any other declination other than 0 produces a hyperbolic line.

This dial used empirical methods to draw the north south meridian line, albeit using calculated time shifts for the EOT and longitude differences.

Calculations were used for the equinox point, and the day's spot was cross checked with its calculated position on the meridian (north south) line.

Hour lines were drawn using calculated hour point distances along the equinoctial line, and angles using inverse hour line angles, those all being calculated distances and angles.
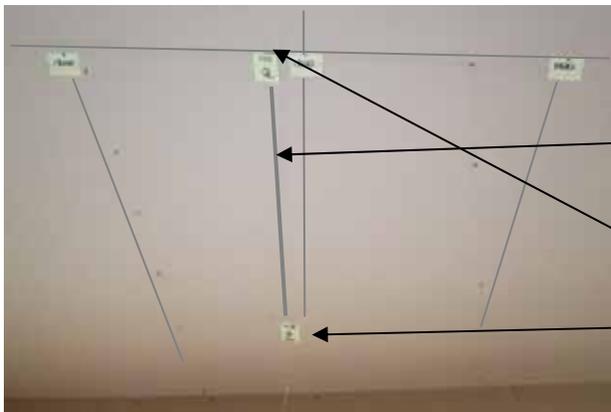
1300, 1200, and 1100 all marked

meridian line

equinox point on the meridian line

summer solstice on the meridian line

reflected light at 10.5 minutes to 1100 which for the December 1 2008 date and the astronomical EOT, was correct.

All required information is provided by:

The main spreadsheet: **illustrating-shadows.xls**
The PDA spreadsheet: a3.2 ceiling data.xls
And a tabular spreadsheet: sheet-a3.2-ceiling-data.xls
And a DeltaCAD program, choice 7:   MAIN-h-dials[f].bas

The main spreadsheet also has within it an almanac and a sun's transit table. The transit table only needs the longitude correction to be converted to minutes and added (if west of the meridian) or subtracted (if east of the meridian). The longitude difference is multiplied by 4 to convert from degrees to minutes.

NOON TRANSIT FOR A SPECIFIED LOCATION				hhmm.m								
Longitude of design dial:			0.00	Longitude of standard time:		0.00		Diff in degrees:		0.00	mins:	0

	1	2	3	4	5	6	7	8	9	10	11	12
	Jan	Feb	Mar	Apr	May	Jun	Jly	Aug	Sep	Oct	Nov	Dec
1	1203.31	1213.32	1212.27	1204.01	1157.50	1157.85	1203.45	1206.22	1200.12	1149.92	1143.74	1148.89
2	1203.78	1213.40	1212.16	1203.44	1157.43	1157.94	1203.56	1206.18	1159.93	1149.73	1143.72	1149.52
3	1204.25	1213.47	1212.03	1203.26	1156.96	1158.43	1204.08	1206.14	1159.74	1149.54	1143.71	1149.75
4	1204.71	1213.53	1211.50	1203.09	1156.90	1158.53	1204.19	1206.09	1159.54	1148.95	1143.71	1149.99
5	1205.16	1213.59	1211.37	1202.51	1156.85	1158.64	1204.29	1206.03	1158.94	1148.76	1143.72	1150.63
6	1205.60	1214.04	1211.24	1202.34	1156.80	1158.74	1204.40	1205.57	1158.74	1148.58	1143.74	1150.88
7	1206.04	1214.08	1211.09	1202.17	1156.75	1158.85	1204.50	1205.50	1158.54	1148.40	1143.77	1151.54
8	1206.47	1214.11	1210.55	1202.00	1156.71	1158.96	1204.59	1205.43	1157.94	1147.83	1143.80	1151.79
9	1206.89	1214.13	1210.40	1201.44	1156.68	1159.48	1205.08	1205.35	1157.73	1147.66	1143.84	1152.46
10	1207.30	1214.15	1210.25	1201.27	1156.65	1159.60	1205.17	1205.27	1157.52	1147.49	1143.90	1152.73
11	1207.71	1214.15	1210.09	1201.11	1156.63	1159.71	1205.26	1205.18	1156.91	1146.93	1143.96	1153.00
12	1208.10	1214.15	1209.54	1200.56	1156.62	1159.84	1205.34	1205.08	1156.70	1146.78	1144.42	1153.68
13	1208.49	1214.14	1209.37	1200.40	1156.61	1159.96	1205.41	1204.58	1156.49	1146.63	1144.50	1153.96
14	1208.86	1214.13	1209.21	1200.21	1156.60	1200.09	1205.48	1204.47	1155.88	1146.48	1144.59	1154.64
15	1209.22	1214.11	1209.04	1200.10	1156.60	1200.21	1205.55	1204.36	1155.66	1145.94	1144.68	1154.92
16	1209.58	1214.08	1208.47	1159.96	1156.61	1200.34	1206.01	1204.24	1155.45	1145.80	1144.78	1155.61
17	1209.92	1214.04	1208.30	1159.82	1156.62	1200.47	1206.06	1204.12	1154.84	1145.68	1144.90	1155.90
18	1210.25	1213.59	1208.13	1159.68	1156.64	1200.60	1206.11	1203.59	1154.62	1145.55	1145.42	1156.60
19	1210.56	1213.54	1207.55	1159.55	1156.67	1201.13	1206.16	1203.45	1154.41	1145.43	1145.54	1156.89
20	1210.87	1213.48	1207.38	1159.42	1156.70	1201.26	1206.20	1203.31	1153.80	1144.92	1145.68	1157.59
21	1211.16	1213.42	1207.20	1158.89	1156.73	1201.39	1206.23	1203.17	1153.58	1144.82	1145.83	1157.88
22	1211.44	1213.35	1207.02	1158.77	1156.77	1201.52	1206.26	1203.02	1152.97	1144.72	1145.98	1158.58
23	1211.71	1213.27	1206.44	1158.66	1156.82	1202.05	1206.28	1202.47	1152.76	1144.63	1146.54	1158.88
24	1211.97	1213.18	1206.26	1158.54	1156.87	1202.18	1206.30	1202.31	1152.55	1144.55	1146.71	1159.58
25	1212.21	1213.09	1206.08	1158.44	1156.92	1202.31	1206.31	1202.15	1151.94	1144.47	1146.88	1159.87
26	1212.44	1212.60	1205.50	1157.94	1156.98	1202.44	1206.32	1201.59	1151.73	1144.00	1147.47	1200.17
27	1212.65	1212.49	1205.32	1157.84	1157.45	1202.56	1206.32	1201.42	1151.52	1143.94	1147.66	1200.47
28	1212.86		1205.14	1157.75	1157.52	1203.09	1206.31	1201.24	1150.92	1143.88	1147.86	1201.16
29	1213.04		1204.55	1157.66	1157.59	1203.21	1206.30	1201.07	1150.72	1143.83	1148.46	1201.45
30	1213.22		1204.37		1157.67		1206.28	1200.49		1143.79		1202.14

BLACK	time is at or after legal noon standard time [no summer time correction]		If a leap year then after February
BLUE	before legal noon standard time [no summer time correction]		28 use the next day's value

Thus using the November 30th example, the noon transit time for this location (longitude 108.2) and for this time zone (105) is 3.2 degrees, or 12.8 minutes. The 1148.46 and 12.8 thus added become 1201.6 hhmm.m or 1201:36 as hhmm:ss which is within a few seconds of the calculation for the meridian earlier, the above table for transit time is a middle of the four year span using an astronomically correct EOT, thus the 12 seconds difference is surprisingly accurate..
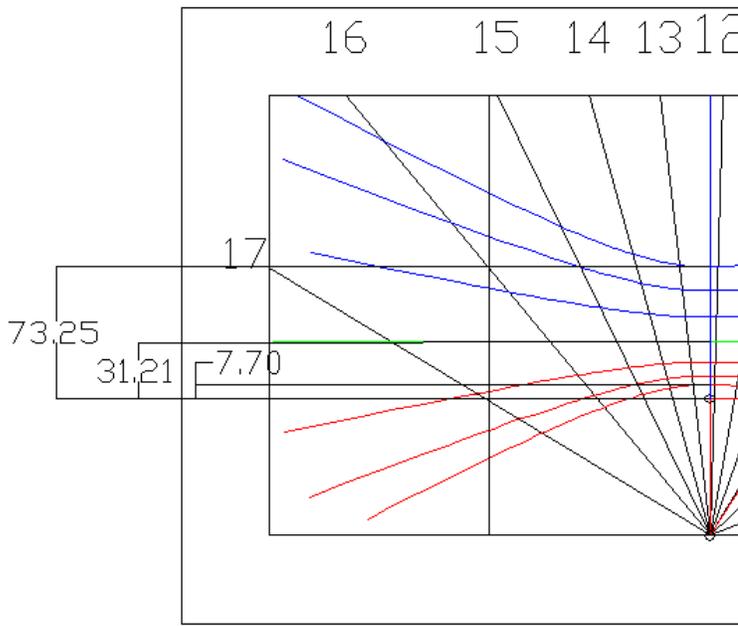
AN EXAMPLE OF THE DELTA CAD MACRO FOR CEILING DIALS
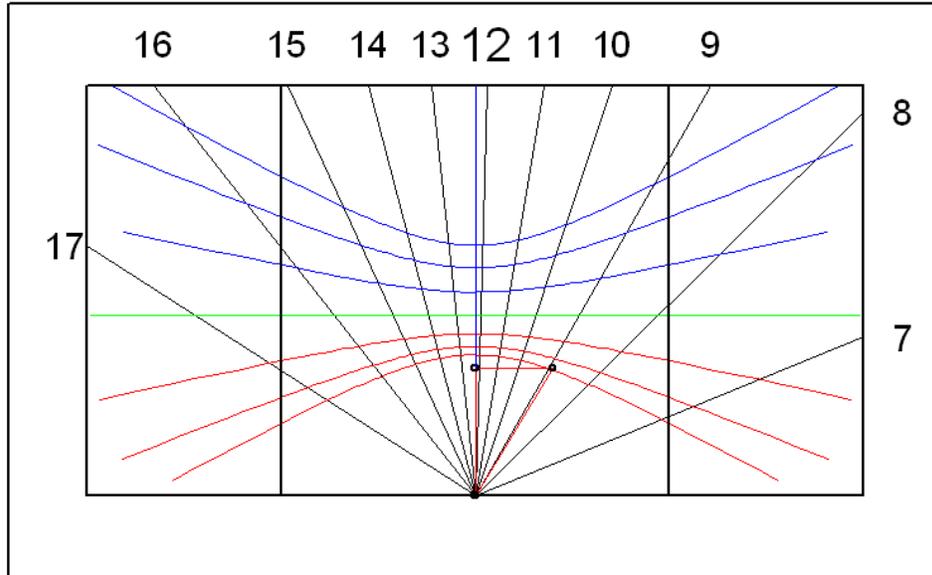
| Ceiling dial | | | | | | Lat: 32.8 d.Long: 03.2 | | | | |
| | | | | | | mirror:ceiling: 48.5 | | | | |

	17	16	15	14	13	12	11	10	9	8	7
hr angle w dial ctr	-58.7	-39.6	-25.8	-15.3	-6.4	01.7	10.1	19.5	31.2	47.0	68.9
angle w equnx line	31.3	50.4	64.2	74.7	83.6	88.3	79.9	70.5	58.8	43.0	21.1

distance above mirror to equinox	031.20
distance above mirror to winter solstice	072.42
distance above mirror to summer solstice	007.95
distance above mirror to dial center	106.60

CALENDAR CURVES AND THE CEILING DIAL (or large horizontal)
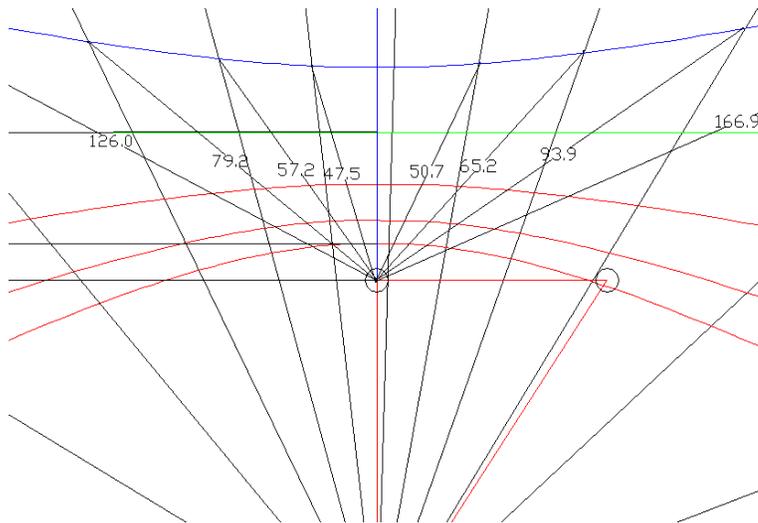
The CAD program can be used to measure distances from the point on the ceiling above the mirror to calendar points on the hour lines.

To the left are CAD measurements of the ceiling point above the mirror to the summer and winter solstices, and the equinox, on the north south meridian (not the noon line).

The measurements are less than an inch off which is fair given the size of the dial plate.

Measurements for each calendar curve are made, in the case to the left, the winter solstice was used.

These distances allow a point to marked on each hour line for the several calendar curves, thus allowing the calendar curves to be drawn.

FINAL DIAL COMPLETION FOR APPEARANCE

Wood molding was used to mark the significant points, wood trim was used for the hour lines and for calendar curve segments. And a stucco used to fill in the areas.

The mirror was enhanced, this was not simple because of the need for perfect alignment with the prior mirror so that the hour lines and calendar curves would still be accurate.

The plan was simple. A laser was set up to send a dot of light off the original mirror and mark that point on the ceiling. Then the old mirror would be removed and replaced with the one to the right. The new mirror would be fine tuned so that the spot would be at the same place as with the original mirror. This works because of the law of reflection.

The laser was set up, and work resumed on the wood for the dial plate borders, and trim for the hour lines and calendar curves.

At this point, some wood fell down and that was the end of the original mirror.

What to do?

The benefits of that original meridian line sprang to mind. Not only does the meridian line indicate the deviation of the walls of the building housing the ceiling dial, not only does the meridian line indicate the central point for the calendar curves, not only does it provide the line from which the equinoctial line stems perpendicular to it, but it also provides the means of calibrating a mirror.

If the sun's noon transit is measured, using the noon transit data from the spreadsheet, or from the time of noon corrected for EOT and longitude, then at noon the sun must be reflected onto that north south meridian line.

And since the distance from the mirror to the ceiling is known, and the day's solar declination, then the distance from the ceiling point above that mirror, to the center of the sun's reflected spot on the ceiling can be deduced.

The ceiling dial spreadsheet has this table in one degree steps. And for a distance of 48.5, and a solar declination of -23.13 shows the spot to be 71.58 inches from the reference point.

DECLINATION POINTS ALONG THE MERIDIAN AS MEASURED FROM THE CEILING POINT ABOVE THE MIRROR FOR A LINEAR DISTANCE OF:			48.50
-23.44	72.42	25-Dec	
-23	71.23		
-22	68.63		today's decl:
-21	66.15		-23.13
-20	63.78		today's spot
-19	61.52		71.58
-18	59.36		

Thus tragedy was averted.

This was not the first tragedy to strike the project. Another tragedy was stepping backwards off the step ladder while marking the ceiling. Serious injury was averted thanks to training I had received in the military in how to jump or fall from a moving vehicle. When working with ladders it is easy to become disoriented, especially when wearing glasses.

Wood markers were applied to the nodes of the dial plate.

Hour lines and calendar curves were completed with 1x2 and with trim. A boundary was made, also with 1x2 wood.

Not shown above, but an Italian 3 hours to sunset line was added, and markers on the meridian line for 5, 10, 15, and 20 degrees of declination.

The ceiling between all the markers and wood molding was textured.

After the dial was constructed, the area was textured with stucco patch.