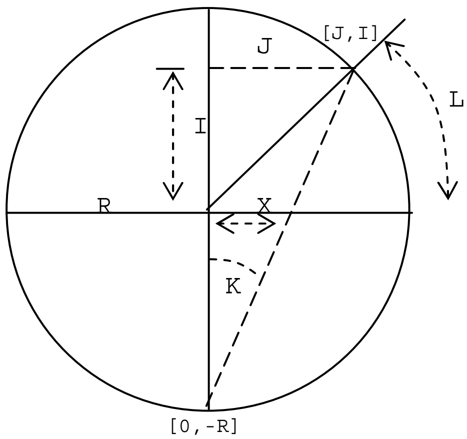
FORMULA DERIVATION FOR PLANISPHERIC ASTROLABES

The geometry of the astrolabe (see chapter 21) can be converted to trigonometry, as follows.



J is the "x" distance from the vertical to the angle "L" intercept point

I is the "y" distance for that intercept point

L is an angle (in real life it is latitude plus as well as minus 90-alt where alt is the suns altitude

R is the "Earth radius" used for the projection

K is an intermediate working angle

X is the distance along the horizontal for the intercept between point J, I and 0,-R

so... given R and L, find X

clearly:	`X = R * tan(K)`
similarly:	`J = (R+I)*tan(K)`
and also:	`I = R * sin(L)`
similarly:	`J = R * cos(L)`
also:	`tan(K) = J / (R + I)`
but:	`X = R * tan(K)    which thus makes    X = (R * J) / (R + I)`

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  = (R * R * cos(L)) / (R + R * sin(L))
  = (R * cos(L)) / (1 + sin(L))
```

The above is the basis for almucanter projections. In which case angle L is the latitude plus and also minus the co-altitude (sun's altitude from 90 degrees). The above is also used for one of the two points of the ecliptic circle, namely the one where L = 23.44 degrees above the horizontal line. So, angle L is derived twice for each altitude of the sun:-

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L = latitude + (90-altitude)    and
L = latitude - (90-altitude)
```
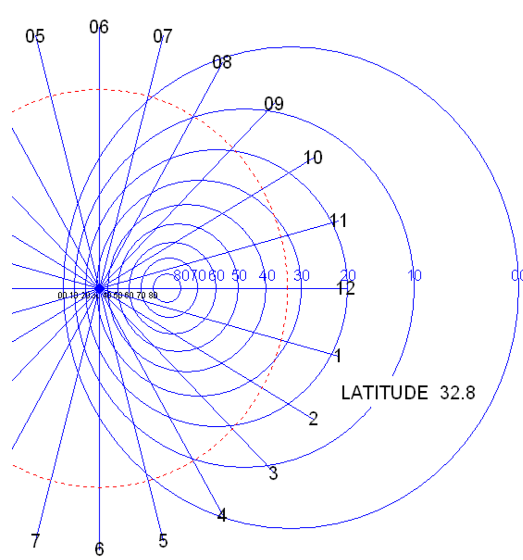

And this generates two X values, from whence we derive a center and a radius for the almucanter (for that altitude at that latitude), namely X1 and X2. The almucanter center and radius are also simply found by:-

the center "C" is at	`y=0 and` `x=0.5*(x1+x2)`
the radius is	`r=x2-C`

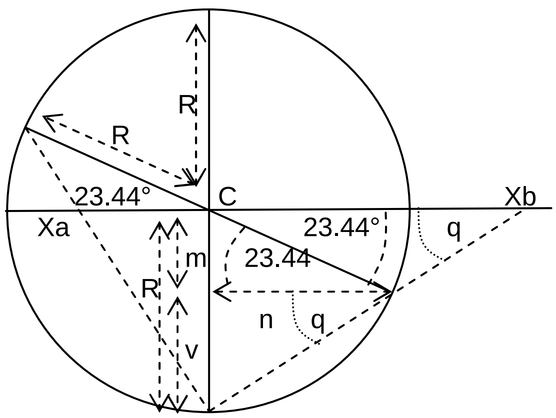


Since we have the latitude, we run a loop from 0 to 90 altitudes and draw those circles.

The detailed logic and coding is contained in the DeltaCAD macro for the astrolabe that is on the CD and web site associated with this book.

A boundary circle is defined by the rete's maximum dimension, XB, see next page.

The ecliptic circle is projected to the rete.



The rete is a circle, that has a rotational center at C but whose circle center is between points Xa and Xb. The rotational center of the rete fits on the almucanter that matches the dial's latitude, i.e. for a dial for latitude 32° the rete's rotational center would be on the 32° almucanter.

Point Xa is found using the formula on the preceding page, using an angle "L" of 23.44°

The same radius used for almucanters must be used for this ecliptic rete also.

"q" is some intermediate angle.

The length from C to point Xb, is found as follows. For simplicity we shall say Xb is the length in question. The angle "q" exists in two places, the vertical drop "m" is in two places also, and radius "R" is on the four radials, the replication of "m" and "R" is not shown to de-clutter the pictorial.

clearly:	sin(23.44)	= m / R	thus:	m = R * sin(23.44)
	cos(23.44)	= n / R	thus:	n = R * cos(23.44)
	tan(q) =	R / Xb	thus:	Xb = R / (tan(q)
			also:	tan(q) = v / n
	v = R – m		thus:	v = R – R * sin(23.44)

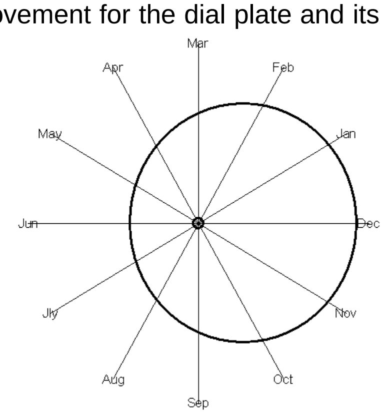
because: tan(q) = v / n then: $\tan(q) = \frac{R - R * \sin(23.44)}{R * \cos(23.44)}$

since: Xb = R / tan(q) then: $Xb = \frac{R}{(R - R * \sin(23.44) / R * \cos(23.44)}$

or: $Xb = \frac{R * R * \cos(23.44)}{R - R * \sin(23.44)}$

or: $Xb = \frac{R * \cos(23.44)}{1 - \sin(23.44)}$

All that is then left is the derivation of the rete's rotational point (0,0), which is also the center from which the month lines radiate, and the rete's physical center for a boundary circle. Xb is the rotational maximum radius, which defines the limiting circle of movement for the dial plate and its almucanters.



It is critical that the "earth radius" be the same for the dial plate and rete, and the DeltaCAD program defaults to an Earth radius of 1 because the end results are scaleable.

While the DeltaCAD drafts are usable, they are included merely to show the mathematical methods involved. There are additional programming needs such as correctly locating the rotational points and the perimeter circles, the DeltaCAD macro offers some insights into those issues.