SUNS DECLINATION FOR ANY GIVEN DAY OF THE YEAR

Source: http://eande.lbl.gov/Task21/C2/algo1/1-11.html

Day number, J J=1 on 1 January, J=365 on 31 December. February is taken to have 28 days.

Jan	*Feb*	*Mar*	*Apr*	*May*	*Jun*
0	*31*	*59*	*90*	*120*	*151*
Jly	*Aug*	*Sep*	*Oct*	*Nov*	*Dec*
181	*212*	*243*	*273*	*304*	*334*

Day angle: da = 2 * pi * (j-1) / 365 (in radians, is an intermediate figure)

Sun Declination: decl = degrees (0.006918 – 0.399912*cos(da) + 0.070257*sin(da)
– 0.006758*cos(2*da) + 0.000907*sin(2*da)
– 0.002697*cos(3*da) + 0.001480*sin(3*da)

SUNS ALTITUDE AND AZIMUTH ON ANY GIVEN HOUR GIVEN THE SUNS DECLINATION

Source: The sun's declination is the value "decl" in the calculations above.
Useful for the shepherd's dial.
Waugh p92, Waugh ch15 p139, Mayall p243
Sundials Australia, Folkard and Ward page 74

ALTITUDE: The sun's altitude is its angle when looked at face on in degrees
Waugh ch15 p139
alt = degrees(ASIN(SIN(decl) * SIN(lat) + COS(decl) *
COS(lat)*COS(lha)))

The suns azimuth: azi = ATAN(SIN(lha) / ((SIN(lat) * COS(lha))
– (COS(lat) * TAN(decl)))
Waugh ch15 p139
Folkard and Ward page 74

note: Waugh presents two different formulae. The one on page 139 agrees with Folkard and Ward, and the one on page 92 does not, but agrees in all aspects except for 6am and thus also 6pm, when using the author's spreadsheet. Similarly, Mayall's formula on page 243 has the same 6am and 6pm anomaly. The Waugh page 139, and Folkard & Ward page 74 seems to work best with the author's spreadsheet.

SUNRISE AND SUNSET TIME FORMULA

Source: Mayall page 243, Rohr page 91, sun's declination is "decl" above

Azimuth of rising/setting sun: A = 180 – arccos (sin(decl) / cos(lat))
Hour angle of rising/setting sun: hsr = arccos(tan(lat) * tan(decl)) from noon

EQUATORIAL DIAL CALENDAR AND SUNRISE/SUNSET LINE DATA

Calendar lines are arcs whose radius = gnomon linear height / tan(declination)
Horizontal sunrise/set line from gnomon distance = gnomon linear height * tan(latitude)

HORIZONTAL DIAL HOUR LINE ANGLE

Proof contained in this book

H = atan (sin(lat) * tan (lha))

POLAR DIAL

Proof contained in this book   sh = style linear height, times are from noon

from style to hour line = sh * tan(lha from noon)
distance up an hour line to a calendar line = sh * tan (declination) / cos (time)

EAST OR WEST VERTICAL HOUR LINES NON DECLINER (MERIDIAN DIAL)

Proof contained in this book   sh = style linear height, times are from 6am or 6pm

from style to hour line = sh * tan(lha from 6pm or 6am)
distance up an hour line to a calendar line = sh * tan (declination) / cos (time)

SOUTH VERTICAL NON DECLINER HOUR LINE ANGLE

Proof contained in this book

x = atan (tan (lha) * sin (90 - lat)) angle hour line makes with 12 o'clock line

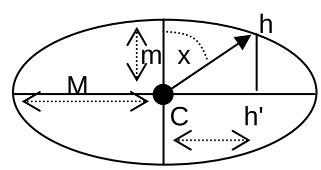
x = atan (tan (lha) * cos (lat))   proof contained in this book

ANALEMMATIC DIAL

Source: Waugh chapter 13, and Rohr chapter 6

m = M sin (Ø)   semi minor axis is sin of latitude times major axis

Ø means latitude
to get an hour point's horizontal distance:-
Ch' = M * sin (15 * hours from noon)
to get an hour point's vertical distance:-
hh' = M * sin (Ø) * cos (15 * hours…)



or to get an hour point by angle from C assuming an ellipse has been drawn
x = arctan (tan(15 * hours…) / sin (Ø)   )
to get the analemmatic points for the gnomon up or down the "m" scale
z = M * tan (dec) * cos (Ø)

STANDARD TIME FROM LAT (Local Apparent Time)

legal time or standard time	= LAT	+ EOT.corr	+ west.long.corr – east.long.corr	+ 1 if summer

STANDARD TIME TO MARK AN HOUR LINE (as in calibrating hour lines using empirical dialing)

dial hour point	=	clock time for that hour point	+ EOT	i.e.
legal time to mark	=	desired LAT	+ EOT	

This is not inconsistent. If the EOT were –10 minutes, then when the dial reads 1400, the legal time would be 1350. So, at 1350, with an EOT of – 10, the sun's shadow will indicate the 1400 hour point. This may appear inconsistent with the rules of algebra, however it is correct. Because of this apparent inconsistency, the dialist is advised to draft a table of times and the hour point they would thus indicate before marking a dial empirically.

SOLAR TIME FROM STANDARD TIME (as in finding true north)

Legal time	= LAT	+	(	+ west.long.corr – east.long.corr	+ EOT.corr	+	1 if summer	)

Solar noon indicates true north because the sun is at its highest point. Thus the shadow produced at solar noon will point to true north. Solar noon happens at the standard time adjusted as follows:-

legal time for solar noon	= 12:00:00 + EOT +1 (if summer) + longitude correction

This is not inconsistent. It may appear that signs should be reversed, however we are in fact achieving the correct arithmetic rules.

VERTICAL DECLINER FACING GENERALLY SOUTH ~ HOUR ANGLES & GNOMON ANGLES

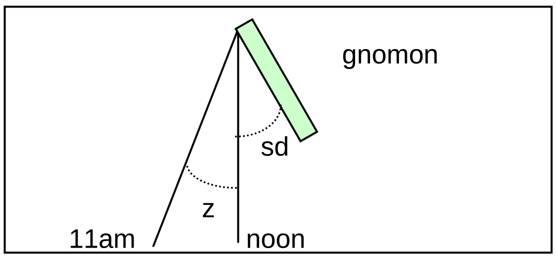
The hour line angles are based on: z = atan(cos(lat)/(cos(dec) cot(ha) + sin(dec) sin(lat))

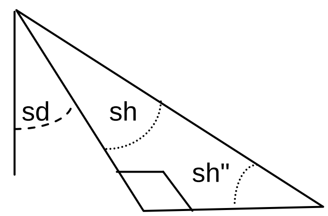
Gnomon rotation or slewing is optional and if used employs the following formula:-

Gnomon offset from vertical is: **sd** = atan(sin(dec) / tan(lat)) **Style Distance**

Style and sub style angle is: **sh** = asin(cos(lat) * cos(dec)) **Style Height**
Style to horizontal angle is: sh" = 90 – sh

Difference in longitude is: dl = atan (tan(dec) / sin(lat)) Diff in Longitude





These formulae are tabulated in the appendices for several latitudes.
Source: Rohr ch 3, page 56, 62, Waugh ch 10 p78, Mayall ap 1 p 237 and in this book.

BIFILAR SUNDIAL

There are variations on this dial design that make it universal. However, the following two formula are for the bifilar dial designed for a specific latitude.

The north south wire can be any height, the east west wire height is equal to:-
east west wire height = height of the north south wire * sin(latitude)

While the north south wire is placed over the noon line, the east west wire is placed at a distance from the dial center that is equal to:-
dist from dial center = height of the north south wire * cos(latitude)

reference: http://www.de-zonnewijzerkring.nl/eng/index-bif-zonw.htm
Beware that the shadow of the cross-hair can be off the dial plate for early hours or winter hours.

EQUATION OF TIME

A formula derived from Frans Maes from data by Savoie producing the EOT in minutes and using two sine waves is used for some spreadsheets, e.g. A2.1b, A2.1c. The values in the sin(...) function result in radians, so the formula is spreadsheet ready as-is. Value d = 1 to 365

E = -7.36*sin(2*3.1416*(d-4.21)/365) + -9.92*sin(4*3.1416*(d-9.9)/365)

Another formula using the sum of three sine waves is used for some spreadsheets, e.g. A2.1d, A2.1e. The sin(...) values result in degrees hence the required indicated radian conversion.

E = -1*(9.84*SIN(RADIANS(2*(360*(mm1+dd-81)/365))) -7.53*COS(RADIANS(360*(mm1+dd-81)/365)) - 1.5*SIN(RADIANS(360*(mm1+dd-81)/365)))-0.3

where: mm1 is the number of days prior to this month's day 1, So Jan is 0, Feb is 31, Mar is 59, April is 90, etc, assuming a non leap year. For leap years add 1 for March to December.

Jan	Feb	Mar	Apr	May	Jun	Jly	Aug	Sep	Oct	Nov	Dec
0	31	59	90	120	151	181	212	243	273	304	334

dd is the day of the month, being 1 to 31

Another three wave formula is:

E = 7.5*SIN(RADIANS(jd-5))-10.2*SIN(RADIANS(1.93*(jd-80)))+0.5*SIN(RADIANS(1.5*(jd-62)))

Another formula derived from the work of Frank Cousins uses the sum of seven sine waves, produces the EOT in seconds, however this book does not use it in any spreadsheets-

E=-(-97.8*SIN(SL)-431.3*COS(SL)+596.6*SIN(2*SL)-1.9*COS(2*SL)+4*SIN(3*SL)+19.3*COS(3*SL)-12.7*SIN(4*SL))

where "SL" is the solar longitude, being SL=(-1*((356/365.2422)*360-270)) + julian day of year
the values in sin(...) result in degrees, so the RADIANS( ) function (not shown) is required for a spreadsheet.

Every approximation is just that, and this book uses several methods for the EOT to demonstrate the real world of approximations, with their benefits as well as drawbacks. Even established published tables vary by almost a minute. Part of this is explained by the year within a leap year cycle, part by the decade the table was printed, and so on.

The most accurate formulae use the astronomical Julian day.

ASTRONOMICAL FORMULAE FOR EOT

The most accurate formulae use astronomical elements. The astronomical Julian day is first calculated, and then other elements build up to a highly generalized EOT formula. "Astronomical Formulae for Calculators" by Jean Meeus is referred to below, fourth edition, ISBN 0-943396-02-6. It's page 24 derives the Julian Day, page 90 derives the equinoxes and solstices for a given year. Do not mix formulae among different books, they may use different baseline epochs, these formulae use Jan 1, 1900 as their epoch, however the formulae work back a couple of thousand years and well into the future. The Julian day discussed here is noon at Greenwich, England.

Julian Day

```
=INT(365.25*(4716+(IF((IF(MM>2,1,0))=0,YYYY-1,YYYY)))
+INT(30.6001*((IF((IF(MM>2,1,0))=0,MM+12,MM))+1))) +DD -1524.5
+(2-INT((IF((IF(MM>2,1,0))=0,YYYY-      1,YYYY))/100)
+INT(INT((IF((IF(MM>2,1,0))=0,YYYY-1,YYYY))/100)/4))
```

where: yyyy = eg 2005, mm=01 to 12, and dd=01 to 31

March equinox: =1721139.2855+365.2421376*YYYY+0.0679190*ZZ*ZZ-0.0027879*ZZ*ZZ*ZZ

June solstice: =1721233.2486+365.2417284*YYYY-0.053018*ZZ*ZZ+0.009332*ZZ*ZZ*ZZ

September equinox: =1721325.6978+365.2425055*YYYY-0.126689*ZZ*ZZ+0.0019401*ZZ*ZZ*ZZ

December solstice: =1721414.392+365.2428898*YYYY-0.010965*ZZ*ZZ-0.0084885*ZZ*ZZ*ZZ

where yyyy = eq 2005, and zz = yyyy/1000

Page 79 provides four ingredients, T, L, M, e. Page 81 provides another two, Obliq and "y". Page 91 deriving the final EOT which is the astronomically accurate EOT in radians, which you convert to degrees, then hours and minutes.

T =(jd-2415020)/36525 A date conversion for the Jan 1, 1900 epoch

L =279.69668+(36000.76892*T)+(0.0003025*T*T)
Geometric mean longitude of the sun

M =358.47583+(35999.04975*T)-(0.00015*T*T)+(0.0000033*T*T*T)
Sun mean anomaly

E =0.01675104-(0.0000418*T)-(0.000000126*T*T)
Earth eccentricity

Obliq =23.452294-(0.0130125*T)-(0.00000164*T*T)+(0.000000503*T*T*T)
Ecliptic obliquity

Y =TAN(RADIANS(OBLIQ/2))*TAN(RADIANS(OBLIQ/2))

EOT =(Y*SIN(RADIANS(2*L))) - (2*E*SIN(RADIANS(M)))+
(4*E*Y*SIN(RADIANS(M))*COS(RADIANS(2*L)))-
(0.5*Y*Y*SIN(RADIANS(4*L))) - ((5/4)*E*E*SIN(RADIANS(2*M)))

mm.mm EOT is the EOT above in radians converted to degrees, divided by 15, and multiplied by – 60 (to get from astronomical EOT to sundial EOT).

The above are employed in the EOT spreadsheet used for table A2.1 which has a number of different worksheets and all that is needed is for the year to be entered once. The spreadsheet then provides the EOT for that year, the year's Julian day for the solstices and equinoxes, the high and low peak values, as well as a five year review of the EOT for the 15th of the month, and finally a highly detailed daily EOT listing.

DETERMINATION OF NEW & FULL MOON, FIRST AND LAST QUARTER

The following formulae are approximations of approximations, and are drawn from chapter 32 p159 of Astronomical Formulae for Calculators, Jean Meeus. Some of the formulae in that book have been simplified elsewhere in this book. The formulae use an epoch base of 1900 which is somewhat academic, however it emphasizes the need for formulae not to be mixed from author to author, or article to article. Many books use the year 2000 as the epoch base.

Additionally, these figures are for the mean lunar cycle. That is 29 days, 12 hours, 44 minutes, 3 seconds, however interaction with the sun can vary this by about 6 hours on either side.

Intermediate figures

```
M23        =   mm/12
N23        =   (dd/30)/12
O23        =   (mm+N23)/12
P23        =   O23+yyyy
k          =   (P23-1900)*12.3685
int~of~k   =   INT(k)
T          =   k/1236.85
```

Julian date for new moon, first quarter, full moon, third quarter

```
=2415020.75933 + 29.53058868 * (int~of~k + 0.00) + 0.0001178*T*T - 0.000000155*T*T*T
=2415020.75933 + 29.53058868 * (int~of~k + 0.25) + 0.0001178*T*T - 0.000000155*T*T*T
=2415020.75933 + 29.53058868 * (int~of~k + 0.50) + 0.0001178*T*T - 0.000000155*T*T*T
=2415020.75933 + 29.53058868 * (int~of~k + 0.75) + 0.0001178*T*T - 0.000000155*T*T*T
```

Julian date you can uses as a base for the above dates

```
=INT(365.25 * (4716 + (IF((IF(mm>2,1,0))=0, yyyy-1, yyyy))) +
        INT(30.6001*((IF((IF(mm>2,1,0))=0, mm+12, mm))+1))) + dd-1524.5 +
        (2-INT((IF((IF(mm>2,1,0))=0, yyyy-1, yyyy))/100) +
        INT(INT((IF((IF(mm>2,1,0))=0, yyyy-1, yyyy))/100)/4))
```

It is emphasized that these formulae are approximations of approximations, and should be treated in that light. They should get you to within a day of the more accurate approaches.